

$\log_e = \ln$

$X^3 \cdot X^5 = X^8$

$\frac{X^{10}}{X^3} = X^7$

$X^{\frac{e}{r}}$

$\log_3(2x) = 3^2 + \log_3 X$

Properties of Logarithms

$\log_a(uv) = \log_a u + \log_a v$ **Product Property**
 $\ln(uv) = \ln u + \ln v$

$\log_a \frac{u}{v} = \log_a u - \log_a v$ **Quotient Property**
 $\ln \frac{u}{v} = \ln u - \ln v$

$\log_a u^n = n \log_a u$ **Power Property**
 $\ln u^n = n \ln u$

Steps
Strategies when expanding logarithms

- Change radicals to rational (fraction) exponents.
- Expand the multiplication and/or division
- Move the exponent(s) LAST

Example 1: $\log \frac{\sqrt{x}}{y^2} X^{\frac{1}{6}}$
 $= \log X^{\frac{1}{6}} - \log y^2$
 $= \frac{1}{6} \log X - 2 \log y$

$\sqrt{x} = X^{\frac{1}{2}}$
 $\sqrt[3]{x} = X^{\frac{1}{3}}$
 $\sqrt[4]{x} = X^{\frac{1}{4}}$

$\log_4 X^5 = 5 \log_4 X$

$\frac{1}{6} \log X - 2 \log y$

Expand.

2. $\log_3 2/x^3 = \log_3 2 + \log_3 X^3$
 $\log_3 2 + 3 \log_3 X$

3. $\log_{10} 5/x^3/y = \log_{10} 5 + \log_{10} X^3 + \log_{10} y$
 $\log_{10} 5 + 3 \log_{10} X + \log_{10} y$

4. $\log_7 \frac{x^3}{y} = \log_7 x^3 - \log_7 y$
 $3 \log_7 X - \log_7 y$

$3 \log_7 X - \log_7 y$

Expand.

5. $\log_4 \frac{5a^2}{b^5} = \log_4 5 + \log_4 a^2 - \log_4 b^5$
 $\log_4 5 + 2 \log_4 a - 5 \log_4 b$

6. $\ln \frac{3(\sqrt[3]{x}) X^{\frac{1}{3}}}{yz^3} = \ln 3 + \ln X^{\frac{1}{3}} - \ln y - \ln z^3$
 $\ln 3 + \frac{1}{3} \ln x - \ln y - 3 \ln z$

$\ln 3 + \frac{1}{3} \ln x - \ln y - 3 \ln z$

Ex. 7 Expand.

$$\log \frac{\sqrt{3x-5}}{7} = \log \left[\frac{(3x-5)^{1/2}}{7} \right]$$

$$= \log(3x-5)^{1/2} - \log 7$$

$$= \frac{1}{2} \log(3x-5) - \log 7$$

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Ex. 3 Use $\log_3 5 \approx 1.465$ and $\log_3 6 \approx 1.631$ to evaluate the logarithm.

A. $\log_3 \frac{6}{5}$	B. $\log_3 30$	C. $\log_3 36$
$\log_3 6 - \log_3 5$	$\log_3 (5 \cdot 6)$	$\log_3 6^2$
$1.631 - 1.465$	$\log_3 5 + \log_3 6$	$2 \log_3 6$
0.166	$1.465 + 1.631$	$2(1.631)$
	3.096	3.262

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① $\log_4 (5x^4z^2) = \log_4 5 + \log_4 x^4 + \log_4 z^2$

$$= \log_4 5 + 4 \log_4 x + 2 \log_4 z$$

② $\log_3 \left(\frac{4x^2y^4}{m^5n^6} \right) = \log_3 4 + \log_3 x^2 + \log_3 y^4 - \log_3 m^5 - \log_3 n^6$

$$= \log_3 4 + 2 \log_3 x + 4 \log_3 y - 5 \log_3 m - 6 \log_3 n$$