

Properties of Logarithms

Expanding $X^{\frac{e}{r}}$ $\sqrt[3]{X} = X^{\frac{1}{3}}$ $\sqrt[3]{X} = X^{\frac{1}{3}}$

Properties of Logarithms		
NAME OF PROPERTY	PROPERTY	EXAMPLE OF PROPERTY
Product Property	$\log_b(xy) = \log_b x + \log_b y$	$\log_3(5x) = \log_3 5 + \log_3 x$
Quotient Property	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_4\left(\frac{x}{3}\right) = \log_4 x - \log_4 3$
Power Property	$\log_b x^n = n \cdot \log_b x$	$\log_5 x^3 = 3 \log_5 x$

Example 1 Expanding Log Expressions

a. $\log_5(2x^6)$

$$\log_5 2 + \log_5 x^6$$

$$\log_5 2 + 6 \log_5 x$$

c. $\log_2 \frac{x^6 y}{z}$

$$\log_2 x^6 + \log_2 y - \log_2 z$$

$$6 \log_2 x + \log_2 y - \log_2 z$$

b. $\ln \frac{10}{\sqrt{x}} = \ln \frac{10}{x^{1/2}}$

$$= \ln 10 - \ln x^{1/2}$$

$$= \ln 10 - \frac{1}{2} \ln x$$

d. $\log(2\sqrt[3]{x})$

$$\log(2/x^{1/3})$$

$$\log 2 + \log x^{1/3}$$

$$\log 2 + \frac{1}{3} \log x$$

Example 2: Condensing Log Expressions

a. $2\log_3 7 - 5\log_3 x$

$$\log_3 7^2 - \log_3 x^5$$

$$\log_3 \left(\frac{49}{x^5} \right)$$

c. $\frac{1}{3} \log_2 x - 3 \log_2 y$

$$\log_2 x^{1/3} - \log_2 y^3$$

$$\log_2 \sqrt[3]{x} - \log_2 y^3$$

$$\log_2 \left(\frac{\sqrt[3]{x}}{y^3} \right)$$

b. $\frac{1}{2} \log x + \log 4 - \log y$

$$\log x^{1/2} + \log 4 - \log y$$

$$\log \sqrt{x} + \log 4 - \log y$$

$$\log \left(\frac{4\sqrt{x}}{y} \right)$$

d. $3 \ln x - 2 \ln y - 4 \ln z$

$$\ln x^3 - \ln y^2 - \ln z^4$$

$$\ln \left(\frac{x^3}{y^2 z^4} \right)$$

① Move the number in front BACK to an exponent.

② Raise numbers to a power

③ Change fraction exponents back to radicals.

④ Condense to one log.